Analysis of Lossy Multiconductor Transmission Lines using the Asymptotic Waveform Evaluation Technique

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***Abstract*—A method is described for the transient analysis of lossy coupled transmission line networks with nonlinear elements. The method combines the asymptotic waveform evaluation technique with a piecewise decomposition algorithm. Two to three orders of magnitude speedup can be achieved relative to previously published methods with comparable accuracy. The method is useful for delay and crosstalk simulation of high speed VLSI interconnects.**

I. Introduction

Rapid advances in the development of VLSI circuit technology and packaging techniques are yielding larger chips with smaller and faster devices. As a result, the interconnect delay time is often significantly longer than the device switching time. In addition, as interconnection densities and switching speeds increase, the electrical length of interconnects becomes a significant fraction of a wavelength and conventional lumped-impedance models can no longer be used for accurate simulation of delay and crosstalk. Instead a distributed model for the interconnect should be used in this case [1]–[3].

Several methods have been proposed for the analysis of networks which contain coupled transmission lines [4]–[12]. In most cases, the transmission line has been analyzed either by using a series of lumped models in the time domain, or by using the frequency domain transformation. In general, these techniques provide a detailed analysis of delay and crosstalk, but they require more computer time than the circuit designer can normally afford. The analysis time increases rapidly with circuit size and degree of cross-coupling. This creates the need for a computationally less expensive method which can adequately approximate the circuit response.

In this paper we present a new method for the analysis of lossy coupled transmission line networks with linear or nonlinear elements. The method has the following advantages:

1) Two or three orders of magnitude speedup compared to previous methods with comparable accuracy,
2) Can handle general transmission line networks with no topological or electrical constraints.

The proposed method is based on asymptotic waveform evaluation (AWE) [13]–[15] in which the transient response is approximated by matching the initial conditions and the first $2q-1$ moments of the exact response to a q-pole model. Analysis of lossy coupled transmission lines with linear elements is described in [15]. The emphasis of this paper is to extend the AWE method to handle the nonlinear case.

In Section II, a general method based on the modified nodal admittance (MNA) matrix is presented for the formulation of the network equations. Section III-A is a review of the AWE method and its application to the analysis of lossy coupled transmission lines with linear elements. Section III-B describes how AWE can be used to calculate the response of a linear network to an arbitrary piecewise linear signal. Extension of the AWE method to the case of nonlinear elements is described in Section IV. Section V presents a summary of the analysis and its computational efficiency. Some numerical examples are presented in Section VI. Details of the transmission line moment calculations required for the AWE analysis are given in the Appendix.

II. Formulation of the Network Equations

Consider a nonlinear network which contains lumped components and arbitrary linear subnetworks. The linear subnetworks may contain distributed components. Without loss of generality the modified nodal admittance matrix equations of the network can be written in the form [16]

$$C\frac{d}{dt}v(t)+Wv(t)+\sum_{k=1}^{N_s}D_k i_k(t)-e(t)-f(v(t))=0 \tag{1}$$

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where

- $\boldsymbol{v}(t) \in \Re^N$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms.
- $\boldsymbol{C} \in \Re^{N\times N}$ and $\boldsymbol{W} \in \Re^{N\times N}$ are constant matrices with entries determined by the lumped linear components,
- $\boldsymbol{D}_k = [d_{i,j}]$, $d_{i,j} \in \{0,1\}$, $i \in \{1,2,\cdots,N\}$, $j \in \{1,2,\cdots,n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $\boldsymbol{i}_k(t) \in \Re^{n_k}$ the vector of currents entering the linear subnetworks k, into the node space $\Re^N$ of the network π,
- $\boldsymbol{e}(t) \in \Re^N$ is the vector of independent sources,
- $\boldsymbol{f}(\boldsymbol{v})$: $\Re^N \to \Re^N$ is a function describing the nonlinear elements of the network.

The first, second and fourth terms in (1) cover the linear lumped components and independent sources. The fifth term covers the nonlinear components. The third term connects the terminal currents of the linear subnetworks to the rest of the network through the mapping matrix $\boldsymbol{D}_k$.

Let the frequency domain equations of the linear subnetwork k be in the form

$$\boldsymbol{P}_k \boldsymbol{V}_k(s) + \boldsymbol{Q}_k \boldsymbol{I}_k(s) = 0 \tag{2}$$

where $\boldsymbol{V}_k$ and $\boldsymbol{I}_k$ represent the Laplace domain terminal voltages and currents of the subnetwork k, respectively.

In the special case where the subnetwork k consists of a multiconductor transmission line system, $\boldsymbol{P}_k$ and $\boldsymbol{Q}_k$ can be described in terms of the line parameters (Section III-B).

III. Case 1: Lossy Coupled Transmission Lines with Linear Elements

A. Approximating the Impulse Response of a Linear Network

When the network does not contain any nonlinear elements, $\boldsymbol{f}(\boldsymbol{v}) = 0$, the impulse response of (1) can be written in the complex frequency domain in the form

$$\begin{bmatrix} s\boldsymbol{C}+\boldsymbol{W} & \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots & \boldsymbol{D}_{N_s} \\ \boldsymbol{P}_1\boldsymbol{D}_1^t & \boldsymbol{Q}_1 & 0 & \cdots & 0 \\ \boldsymbol{P}_2\boldsymbol{D}_2^t & 0 & \boldsymbol{Q}_2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \boldsymbol{P}_{N_s}\boldsymbol{D}_{N_s}^t & 0 & 0 & \cdots & \boldsymbol{Q}_{N_s} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(s) \\ \boldsymbol{I}_1(s) \\ \boldsymbol{I}_2(s) \\ \cdot \\ \cdot \\ \boldsymbol{I}_{N_s}(s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{3}$$

or

$$\boldsymbol{Y}(s)\boldsymbol{Z}(s) = \boldsymbol{b} \tag{4}$$

where

s is the complex frequency,

$$\boldsymbol{Z}(s) = \begin{bmatrix} \boldsymbol{V}(s) & \boldsymbol{I}_1(s) & \boldsymbol{I}_2(s) & \cdots & \boldsymbol{I}_{N_s}(s) \end{bmatrix}^t,$$

$$\boldsymbol{Y}(s) = \begin{bmatrix} s\boldsymbol{C}+\boldsymbol{W} & \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots & \boldsymbol{D}_{N_s} \\ \boldsymbol{P}_1\boldsymbol{D}_1^t & \boldsymbol{Q}_1 & 0 & \cdots & 0 \\ \boldsymbol{P}_2\boldsymbol{D}_2^t & 0 & \boldsymbol{Q}_2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \boldsymbol{P}_{N_s}\boldsymbol{D}_{N_s}^t & 0 & 0 & \cdots & \boldsymbol{Q}_{N_s} \end{bmatrix},$$

$\boldsymbol{b} \in \Re^N$, $\boldsymbol{b}_i \in \{-1,0,1\}$ is a constant vector with entries determined by the independent sources.

To approximate the impulse response $z(t)$ using the asymptotic waveform evaluation technique (4) is expanded in a Maclaurin's series of the form

$$\boldsymbol{Z}(s) = \sum_{n=0}^{\infty} \boldsymbol{M}_n s^n \tag{5}$$

where

$$\boldsymbol{M}_n = \frac{\dfrac{\partial^n}{\partial s^n}|[\boldsymbol{Y}^{-1}\boldsymbol{b}]|_{s=0}}{n!}. \tag{6}$$

The moments $m_n^i = [M_n]_{(i)}$; $n = 0,1,2,\ldots,2q-1$ of an output i are then matched to a lower order frequency domain function in the form

$$[Z^*(s)]_{(i)} = \sum_{j=1}^{q} \frac{k_j^i}{s - p_j^i}. \tag{7}$$

The approximate time domain transient solution is then

$$[z^*(t)]_{(i)} = \sum_{j=1}^{q} k_j^i e^{p_j^i t}. \tag{8}$$

Given the moments $[M_n]_{(i)}$, evaluation of the poles p_j^i and the residues k_j^i; $j = 1,2,\cdots,q$ is described in details in [13]–[14].

Using (4) and (6), a recursive equation for the evaluation of the moments can be obtained in the form

$$[\boldsymbol{Y}]^{(0)}\boldsymbol{M}_n = -\sum_{r=1}^{n} \frac{[\boldsymbol{Y}]^{(r)}\boldsymbol{M}_{n-r}}{r!} \tag{9}$$

where

$$[\boldsymbol{Y}]^{(0)} = \boldsymbol{Y}(s=0),$$

$$[\boldsymbol{Y}]^{(r)} = \left[\frac{\partial^r}{\partial s^r}\boldsymbol{Y}(s)\right]_{s=0}$$

with

$$[\boldsymbol{Y}]^{(0)}\boldsymbol{M}_0 = \boldsymbol{b}. \tag{10}$$

Next the derivatives of Y are obtained as

$$[Y]^{(1)} = \begin{bmatrix} C & 0 & 0 & \cdots & 0 \\ P_1^{(1)} D_1^t & Q_1^{(1)} & 0 & \cdots & 0 \\ P_2^{(1)} D_2^t & 0 & Q_2^{(1)} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ P_{N_s}^{(1)} D_{N_s}^t & 0 & 0 & \cdots & Q_{N_s}^{(1)} \end{bmatrix} \quad (11)$$

and

$$[Y]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ P_1^{(r)} D_1^t & Q_1^{(r)} & 0 & \cdots & 0 \\ P_2^{(r)} D_2^t & 0 & Q_2^{(r)} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & 0 \\ P_{N_s}^{(r)} D_{N_s}^t & 0 & 0 & \cdots & Q_{N_s}^{(r)} \end{bmatrix}; \quad (r \geqslant 2). \quad (12)$$

Evaluation of the derivatives $P_k^{(r)}$ and $Q_k^{(r)}$ is described in [15] for the case where the subnetwork consists of a multiconductor transmission line system. A summary is given in the Appendix.

B. Approximation of the Response of a Linear Network to an Arbitrary Piecewise Linear Input Waveform

The analysis method described in Section III-A provides a closed form approximation, (7), to the Laplace domain impulse response of the network. This form is equivalent to the closed form time domain impulse response given by (8). Network response to a waveform that can be represented in the Laplace domain as a ratio of two polynomials can be calculated by carrying out symbolic multiplication in the Laplace domain and reducing the result to the form of (7). Exact conversion to the time domain is possible using (8). Essentially the linear network is approximated by the impulse response $z^*(t)$ and exact calculations are made using the approximate response and the applied waveforms.

As an example consider an applied ramp waveform. Including the ramp in (7) gives

$$[Z^*(s)]_r = \sum_{j=1}^{q} \frac{k_j}{s^2(s-p_j)} \quad (13)$$

where $[Z^*(s)]_r$ is the desired output response due to the ramp input. Completing the fraction and converting to the time domain yields

$$[z^*(t)]_r = \alpha_0 + \alpha_1 t + \sum_{j=1}^{q} k_j' e^{p_j t} \quad (14)$$

where α_0, α_1 and k_j' are constants.

This method can be extended to arbitrary piecewise linear waveforms as follows. Let $x(t)$ be a piecewise

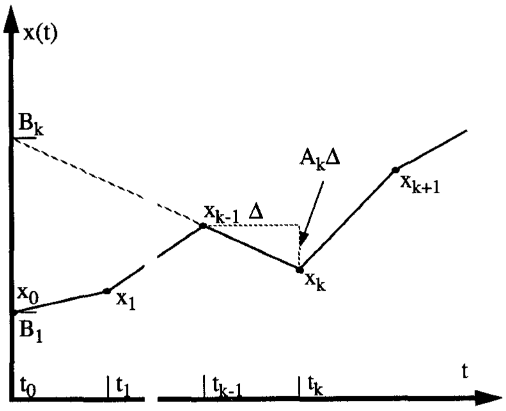

Fig. 1. Piecewise linear function.

linear function (Fig. 1) described by

$$x(t) = \sum_{k=1}^{L} (B_k + A_k t)[u(t-t_{k-1}) - u(t-t_k)] \quad (15)$$

where

L is the number of linear segments,

$$A_k = \frac{x_k - x_{k-1}}{t_k - t_{k-1}}, \qquad B_k = \frac{x_{k-1}t_k - x_k t_{k-1}}{t_k - t_{k-1}},$$

and $u(t)$ is the unit step function.

Divide the interval of analysis $t \in [0,T]$ into L equal length intervals such that $t_k - t_{k-1} = \Delta$ and $L\Delta = T$. Equation (15) may be rewritten as

$$x(t) = \sum_{k=1}^{L} (B_k + A_k t_{k-1}) u_h(t - t_{k-1}) + \sum_{k=1}^{L} A_k u_g(t - t_{k-1}) \quad (16)$$

where

$$u_h(t) = u(t) - u(t-\Delta)$$
$$u_g(t) = t(u(t) - u(t-\Delta)).$$

Equation (16) expresses the piecewise linear function $x(t)$ as a scaled and shifted sum of the functions $u_h(t)$ and $u_g(t)$. The response of a linear network to an input $x(t)$ can then be constructed using the response to the inputs $u_h(t), u_g(t)$ and superposition theory as

$$v(t) = \sum_{k=1}^{L} h(t-t_{k-1})(B_k + A_k t_{k-1}) + \sum_{k=1}^{L} g(t-t_{k-1}) A_k \quad (17)$$

where $h(t)$ is the response of the network to the input $u_h(t)$ and $g(t)$ is the response of the network to the input

$u_g(t)$. Using the method illustrated in (13) and (14):

$$h(t) = a_0[u(t) - u(t-\Delta)] + \sum_{j=1}^{q} k'_j\left[u(t)e^{p_j t} - u(t-\Delta)e^{p_j(t-\Delta)}\right] \quad (18)$$

$$g(t) = b_0[u(t) - u(t-\Delta)] + b_1[tu(t) - (t-\Delta)u(t-\Delta)] + \sum_{j=1}^{q} k''_j\left[u(t)e^{p_j t} - u(t-\Delta)e^{p_j(t-\Delta)}\right] \quad (19)$$

where the additional constants result from the partial fraction expansion. Substituting for A_k and B_k in (17) yields

$$v(t) = \sum_{k=1}^{L} h(t - t_{k-1})x_{k-1} + \sum_{k=1}^{L} g(t - t_{k-1})\frac{x_k - x_{k-1}}{\Delta}. \quad (20)$$

When the system has more than one output, $h(t)$, $g(t)$ and $v(t)$ will be column vectors. For a multiple-input multiple-output system x_k and $v(t)$ will be vectors and $h(t)$ and $g(t)$ will be matrices.

IV. Case 2: Lossy Coupled Lines with Nonlinear Elements

When nonlinear elements are present $f(v) \neq 0$. In this case the nonzero entries in $\boldsymbol{f}(\boldsymbol{v}(t))$ are replaced by a set of independent waveforms $\boldsymbol{y}(t)$ [17] such that

$$\boldsymbol{f}(\boldsymbol{v}(t)) = \boldsymbol{D}_f \boldsymbol{y}(t) \quad (21)$$

where

$$\boldsymbol{y}(t) \in \Re^{N_f}$$

$\boldsymbol{D}_f = [d_{ij}]$, $d_{ij} \in \{0,1\}$, $i \in \{1,2,\cdots,N_f\}$, $j \in \{1,2,\cdots,N\}$ is a selector matrix, and N_f is the number of nonzero entries in $f(v(t))$.

Using (21), (1) is reduced to a set of linear differential equations in the form

$$\boldsymbol{C}\frac{d}{dt}\boldsymbol{v}(t) + \boldsymbol{W}\boldsymbol{v}(t) + \sum_{k=1}^{N_s} \boldsymbol{D}_k \boldsymbol{i}_k(t) - \boldsymbol{e}(t) - \boldsymbol{D}_f \boldsymbol{y}(t) = 0 \quad (22)$$

With reference to Fig. 1, let $\boldsymbol{y}(t)$ be a piecewise linear waveform. As described in Section III-B AWE is used to solve (22) using the waveforms $u_h(t)$ and $u_g(t)$ in place of the input $\boldsymbol{y}(t)$. The network equations will be solved once per interval at the time $t_r^*, t_{r-1} < t_r^* \leqslant t_r$. The response of the linear network will be the sum of the responses due to the independent inputs and the waveforms $\boldsymbol{y}(t)$

$$\boldsymbol{v}_r^* = \boldsymbol{v}(t_r^*) = \sum_{k=1}^{L} \boldsymbol{H}(t_r^* - t_{k-1})\boldsymbol{y}_{k-1} + \sum_{k=1}^{L} \boldsymbol{G}(t_r^* - t_{k-1})\frac{\boldsymbol{y}_k - \boldsymbol{y}_{k-1}}{\Delta} + \boldsymbol{F}(t_r^*) \quad (23)$$

where $\boldsymbol{F}(t)$ is the response due to the independent inputs and $\boldsymbol{G}(t)$ and $\boldsymbol{H}(t)$ are the matrices

$$\boldsymbol{G}(t) = \{g_{ij}\}; \quad i \in \{1,\cdots,N\} \quad j \in \{1,\cdots,N_f\} \quad (24)$$

$$g_{ij}(t) = [\boldsymbol{v}(t)]_i \begin{cases} y_j(t) = u_g(t) \\ y_k(t) = 0, k \neq j \\ e(t) = 0 \end{cases} \quad (25)$$

$$\boldsymbol{H}(t) = \{h_{ij}\}; \quad i \in \{1,\cdots,N\} \quad j \in \{1,\cdots,N_f\} \quad (26)$$

$$h_{ij}(t) = [\boldsymbol{v}(t)]_i \begin{cases} y_j(t) = u_h(t) \\ y_k(t) = 0, k \neq j \\ e(t) = 0. \end{cases} \quad (27)$$

The augmenting waveforms $\boldsymbol{y}(t)$ are computed starting with an initial guess $\boldsymbol{y}^0(t)$ and an iterative technique [3] based on the Newton–Raphson method. Iteration continues until the waveforms $\boldsymbol{y}(t)$ satisfy the linear network constraints (22) and the nonlinear constraints (21).

From (23) v_r^* may be written as an explicit function of y_r:

$$\boldsymbol{v}_r^* = \boldsymbol{R}_r \boldsymbol{y}_r + \boldsymbol{S}_r \quad (28)$$

where

$$\boldsymbol{R}_r = \frac{\boldsymbol{G}(t_r^* - t_{k-1})}{\Delta} \quad (29)$$

$$\boldsymbol{S}_r = \sum_{k=1}^{r}\left[\boldsymbol{H}(t_r^* - t_{k-1}) - \frac{\boldsymbol{G}(t_r^* - t_{k-1})}{\Delta}\right]\boldsymbol{y}_{k-1} + \sum_{k=1}^{r-1}\frac{\boldsymbol{G}(t_r^* - t_{k-1})}{\Delta}\boldsymbol{y}_k + \boldsymbol{F}(t_r^*). \quad (30)$$

Using (28) it is possible to solve for one time point at a time using Newton iterations in the form

$$\boldsymbol{J}^{(i)}\left[\boldsymbol{y}_r^{(i+1)} - \boldsymbol{y}_r^{(i)}\right] = -\left[\boldsymbol{D}_f \boldsymbol{f}(\boldsymbol{v}_r^{*(i)}) - \boldsymbol{y}_r^{*(i)}\right] \quad (31)$$

$$\boldsymbol{J} = \boldsymbol{D}_f\left[\frac{df}{dv}\right]\boldsymbol{R}_r - \frac{t_r^* - t_{t-1}}{\Delta}. \quad (32)$$

V. Computational Considerations

A. Summary of the Computational Steps

There are three separate steps in the analysis:

1) AWE is used to approximate the linear network with a set of poles and residues. Residues must be calculated for the response at each desired output value, and each controlling signal for nonlinear elements (Section III-A).

2) Calculation of the linear network response due to independent sources as well as the sources u_h, and u_g. The approximate poles and residues calculated in the previous stage are combined with exact Laplace domain representations of the input signals. The resulting closed form expression has an exact time domain representation (Section III-B).

3) Iterative solution of the nonlinear equations using superposition of the linear network responses calculated in 2 (Section IV).

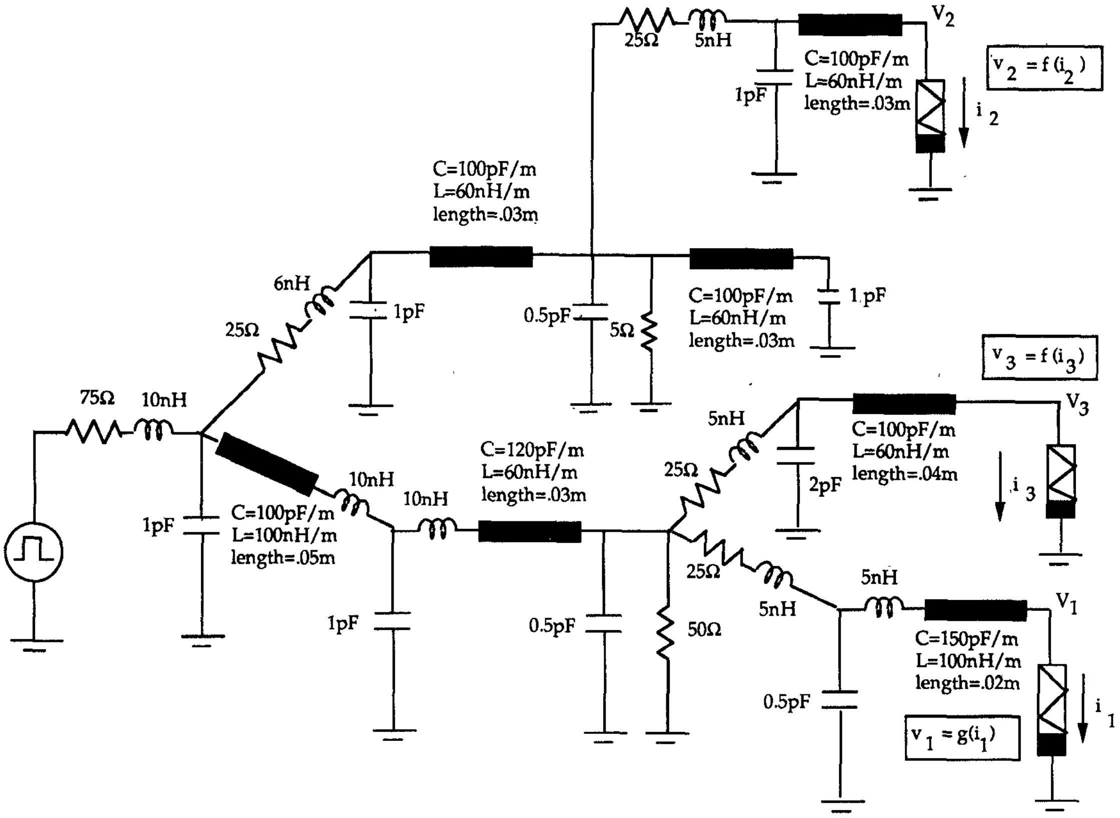

Fig. 2. Network for example 1, interconnect model with transmission lines.

TABLE I
MATRIX OPERATION COUNT COMPARISON OF ANALYSIS ALGORITHMS

Operation \ Method	FFT	NILT	AWE
LU	N_l	MxN_t	1
F/B	$N_i x N_l$	$MxN_i xN_t$	$2xPxN_i$

N_l is the number of frequency points, usually in the range $2000 \leqslant N_l \leqslant 20000$, N_t is the number of time points, usually in the range $50 \leqslant N_t \leqslant 200$, M is the number of poles in the NILT approximation, usually in the range $5 \leqslant M \leqslant 11$, N_i is the number of independent inputs, P is the order of the AWE approximation.

B. Computational Cost

The AWE analysis method requires a single LU decomposition of the MNA matrix. This makes AWE less computational expensive than other comparable methods for obtaining the time domain response of a linear network described in the frequency domain. Table I compares the major matrix operation requirements of AWE analysis with FFT and numerical inversion of the Laplace transform (NILT) [7] for a multiple-input multiple-output system such as a nonlinear network being analyzed by the method described in Section IV. A complex network with many reflections requiring thousands of frequency points for the FFT is assumed. The comparison shows that even for a typical medium sized interconnect network AWE could be hundreds of times faster than the FFT approach. Example 3 compares the run times of the AWE algorithm and HSPICE time domain transient analysis.

AWE requires only a small amount of memory as only the approximate poles and residues need to be stored. In contrast FFT and NILT analysis require storage of the entire frequency or time domain response.

A multiple-input multiple-output linear network has a unique characteristic polynomial shared by all responses. Using the AWE approximation each input-output relationship is approximated by a different set of poles. Although the order of the approximating polynomial is normally low (3rd–7th), the CPU time required to find the poles could contribute a significant portion to the total computational cost. It is possible to use a common set of poles for all or some of the responses reducing the required computations. Three approaches are easily identified:

1) Use one set of poles for all responses.
2) Use one set of poles for each input set.
3) Calculate poles for every response.

The first approach uses a single set of poles for all calculations. This requires one pole finding process. The second approach uses a different set of poles for each input point. Thus the second approach requires $N_f + 1$ sets of poles, where N_f is the number of nonzero entries in $f(v(t))$ as described in (21). The third and most accurate approach requires a different set of poles for each response or $N_o^*(N_f + 1)$ pole sets, where N_o is the number of outputs. The fewer poles used the lower the

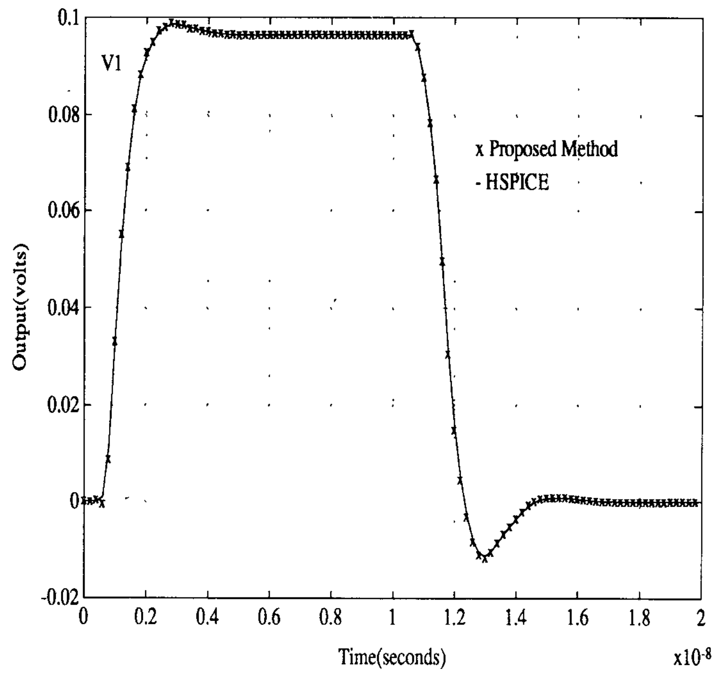

Fig. 3. Pulse response V_1 of the network shown in Fig. 2.

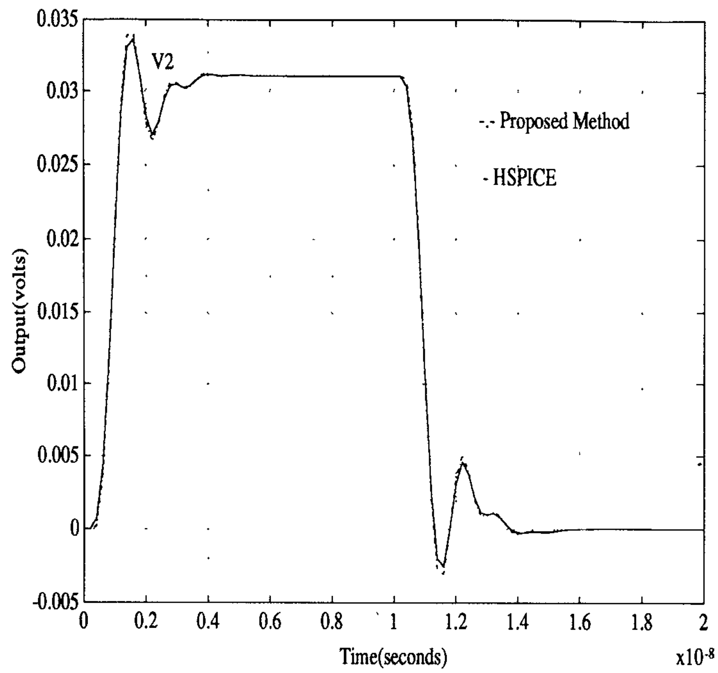

Fig. 4. Pulse response V_2 of the network shown in Fig. 2.

accuracy of the calculated responses. The loss of accuracy is illustrated in Example 4.

VI. Computational Results and Comparisons

Example 1

A multiple transmission line network with nonlinear elements is shown in Fig. 2. The input is a 10 ns pulse with 1 nanosecond rise and fall times. The nonlinear functions are defined as $f(i) = 50i + 21.5i^{1/3}$ and $g(i) = 50i + i^{1/3}$. Output waveforms are shown in Figs. 3–5. As the transmission lines are lossless simulation results can be compared with HSPICE.

Example 2

Consider the network shown in Fig. 6. Both of the transmission lines are 0.1m long. The two conductor line has the following parameters:

$$\boldsymbol{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{nH/m} \qquad \boldsymbol{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{pF/m}$$

$$\boldsymbol{R} = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/\text{m} \qquad \boldsymbol{G} = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \text{S/m}$$

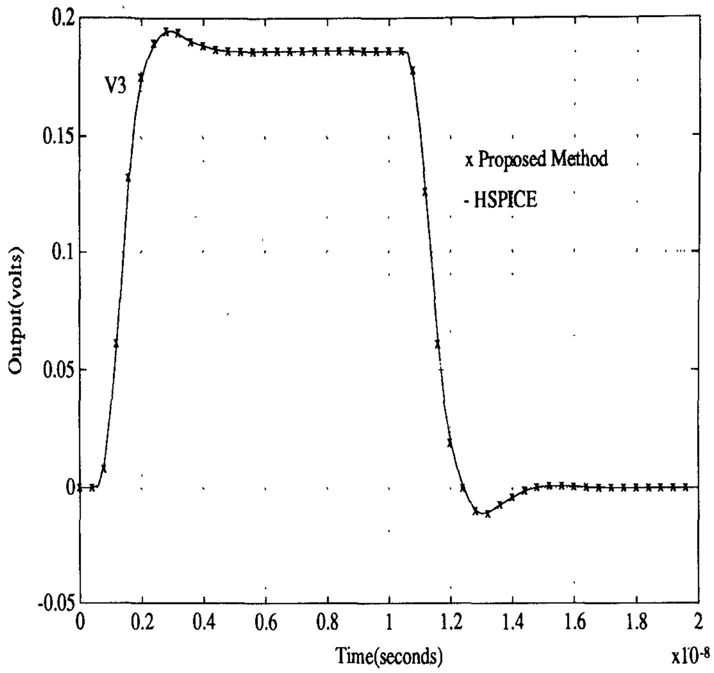

Fig. 5. Pulse response V_3 of the network shown in Fig. 2.

and the parameters of the four conductor line are

$$L = \begin{bmatrix} 494.6 & 63.3 & 7.8 & 0.0 \\ 63.3 & 494.6 & 63.3 & 7.8 \\ 7.8 & 63.3 & 494.6 & 63.3 \\ 0.0 & 7.8 & 63.3 & 494.6 \end{bmatrix} \text{nH/m}$$

$$C = \begin{bmatrix} 62.8 & -4.9 & -0.3 & 0.0 \\ -4.9 & 62.8 & -4.9 & -0.3 \\ -0.3 & -4.9 & 62.8 & -4.9 \\ 0.0 & -0.3 & -4.9 & 62.8 \end{bmatrix} \text{pF/m}$$

$$R = \begin{bmatrix} 50 & 10 & 1 & 0.0 \\ 10 & 50 & 10 & 1 \\ 1 & 10 & 50 & 10 \\ 0.0 & 1 & 10 & 50 \end{bmatrix} \Omega/\text{m}$$

$$G = \begin{bmatrix} 0.1 & -0.01 & -0.001 & 0.0 \\ -0.01 & 0.1 & -0.01 & -0.001 \\ -0.001 & -0.01 & 0.1 & -0.01 \\ 0.0 & -0.001 & -0.01 & 0.1 \end{bmatrix} \text{S/m}$$

Fig. 7 shows the response at node b as calculated using the proposed method, and by numerical inversion of Laplace transformation (NILT) [7]. The applied voltage is a 3 ns pulse with 1 ns rise and fall times.

Example 3

The efficiency of the proposed technique is shown by comparison with HSPICE in Table II. The network of Example 1 with lossless lines was cascaded to achieve the indicated number of transmission lines. HSPICE ran transient analysis with the nonlinear elements modeled by dependent sources. Run times were measured on a SUN 3/60. A speed factor of approximately 45 to 1200 was obtained, depending on the size of the network. Excellent agreement between the two methods similar to that shown in Fig. 4 was obtained for all results.

Example 4

As mentioned in Section V.B it is possible to further reduce computational requirements by reducing the number of pole sets calculated. Fig. 8 illustrates the differences between the three approaches described in Section V-B. It should be noted that while the first two approaches have peak errors around 20% they still give accurate measures of the delay times.

VII. Conclusion

A method has been presented for the analysis of lossy multiconductor transmission line networks with linear or nonlinear elements. The method extends the asymptotic waveform evaluation technique to handle nonlinear components. A piecewise decomposition technique is used in which the nonlinear entries in the network equations are replaced by a set of time-dependent waveforms. The response of the resulting linear network is approximated by matching the first $2q-1$ moments to a lower q-pole model. An iterative technique is described for the evaluation of the parameters defining the augmenting waveforms. The proposed simulation algorithm offers two to three orders of magnitude speedup compared to previously published methods with comparable accuracy and can be used for efficient estimation of delay and crosstalk of high-speed VLSI interconnects.

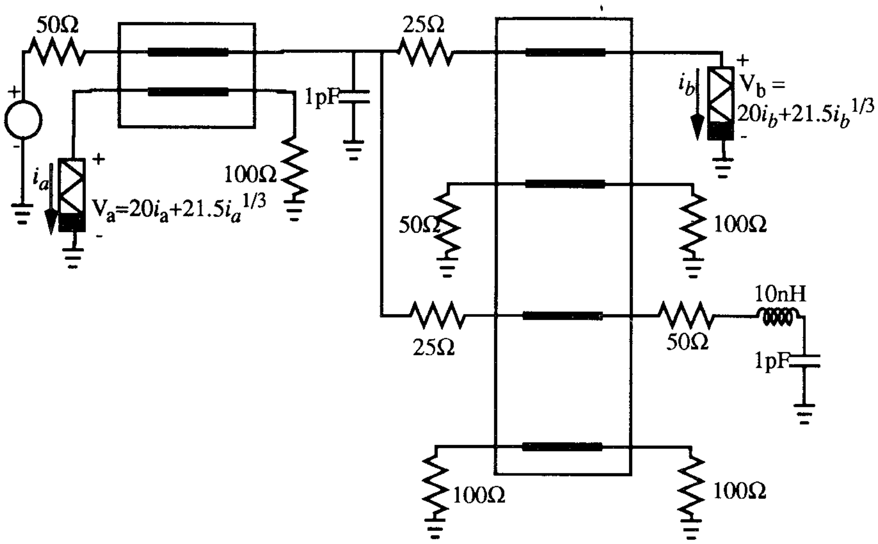

Fig. 6. Network for example 2, interconnect models with lossy coupled transmission lines and nonlinear elements.

TABLE II
CPU TIME COMPARISON

Circuit #	Number of TL	Node Count	Lumped Elements	CPU Seconds PM	CPU Seconds HSPICE
1	35*	101	145	2.62	256
2	35**	101	145	5.61	261
3	105*	301	435	6.05	7702

*linear elements.
**nonlinear elements.
PM-proposed method.

APPENDIX
THE MULTICONDUCTOR TRANSMISSION LINE MOMENTS

Details of the evaluation of the moments can be found in [15]. For the sake of completeness a summary is given here.

For a transmission line uniform along its length with an arbitrary cross section the derivatives $P_k^{(r)}$ and $Q_k^{(r)}$ and hence the moments may be derived from parameters describing the line. The cross section with N signal conductors, can be represented by the following NxN matrices of line parameters: the inductance per unit length $\boldsymbol{L}$, the resistance per unit length $\boldsymbol{R}$, the capacitance per unit length $\boldsymbol{B}$, and the conductance per unit length $\boldsymbol{G}$.

Let γ_m^2 be an eigenvalue of the matrix $\boldsymbol{Z}_L \boldsymbol{Y}_L$ with an associated eigenvector $\boldsymbol{S}_m$, where

$$\boldsymbol{Z}_L = \boldsymbol{R} + s\boldsymbol{L} \tag{33}$$

$$\boldsymbol{Y}_L = \boldsymbol{G} + s\boldsymbol{B}. \tag{34}$$

It can be shown [15] that the terminal voltages and currents are related by (2) where

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{S}_v \boldsymbol{E}_1 \boldsymbol{S}_v^{-1} & -\boldsymbol{U} \\ \boldsymbol{S}_i \boldsymbol{E}_2 \boldsymbol{S}_v^{-1} & 0 \end{bmatrix} \tag{35}$$

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{S}_v \boldsymbol{E}_2 \boldsymbol{S}_i^{-1} & 0 \\ \boldsymbol{S}_i \boldsymbol{E}_1 \boldsymbol{S}_i^{-1} & -\boldsymbol{U} \end{bmatrix} \tag{36}$$

$\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are diagonal matrices defined in terms of the eigenvalues,

$$\boldsymbol{E}_1 = \text{diagonal}\left\{ \frac{\exp(-\gamma_m D) + \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \cdots, N \tag{37}$$

$$\boldsymbol{E}_2 = \text{diagonal}\left\{ \frac{\exp(-\gamma_m D) - \exp(\gamma_m D)}{2} \right\}, \quad m = 1, \cdots, N, \tag{38}$$

where D is the length of the line,
$\boldsymbol{S}_v$ is a matrix with the eigenvectors $\boldsymbol{S}_m$ in the columns,
$\boldsymbol{S}_i = \boldsymbol{Z}_L^{-1} \boldsymbol{S}_v \boldsymbol{\Omega}$,
Ω is a diagonal matrix with $\Omega_{m,m} = \gamma_m$.
From (35) and (36) the moments required in (12) are

$$\boldsymbol{P}^{(r)} = \begin{bmatrix} \left[\boldsymbol{S}_v \boldsymbol{E}_1 \boldsymbol{S}_v^{-1}\right]^{(r)} & 0 \\ \left[\boldsymbol{S}_i \boldsymbol{E}_2 \boldsymbol{S}_v^{-1}\right]^{(r)} & 0 \end{bmatrix} \tag{39}$$

$$\boldsymbol{Q}^{(r)} = \begin{bmatrix} \left[\boldsymbol{S}_v \boldsymbol{E}_2 \boldsymbol{S}_i^{-1}\right]^{(r)} & 0 \\ \left[\boldsymbol{S}_i \boldsymbol{E}_1 \boldsymbol{S}_i^{-1}\right]^{(r)} & 0 \end{bmatrix}. \tag{40}$$

To find a closed-form expression for $\boldsymbol{P}^{(r)}$ and $\boldsymbol{Q}^{(r)}$ Leibnitz's rule is used to expand the derivatives in (39) and (40) in terms of the derivatives of the eigenvectors $\boldsymbol{S}_v$ and the eigenvalues γ_m^2.

Consider $\boldsymbol{S}_v \boldsymbol{E}_1 \boldsymbol{S}_v^{-1} = \boldsymbol{\Phi}$ or $\boldsymbol{S}_v \boldsymbol{E}_1 = \boldsymbol{\Phi} \boldsymbol{S}_v$, differentiation yields

$$\boldsymbol{\Phi}^{(n)} \boldsymbol{S}_v = -\sum_{r=1}^{n} \binom{n}{r} \boldsymbol{\Phi}^{(n-r)} \boldsymbol{S}_v^{(r)} + \sum_{r=1}^{n} \binom{n}{r} \boldsymbol{S}_v^{(n-r)} \boldsymbol{E}_1^{(r)} + \boldsymbol{S}_v^{(n)} \boldsymbol{E}_1. \tag{41}$$

Which gives derivatives for Φ in terms of lower order derivatives and the derivatives of $\boldsymbol{S}_v$ and $\boldsymbol{E}_1$. Differentiat-

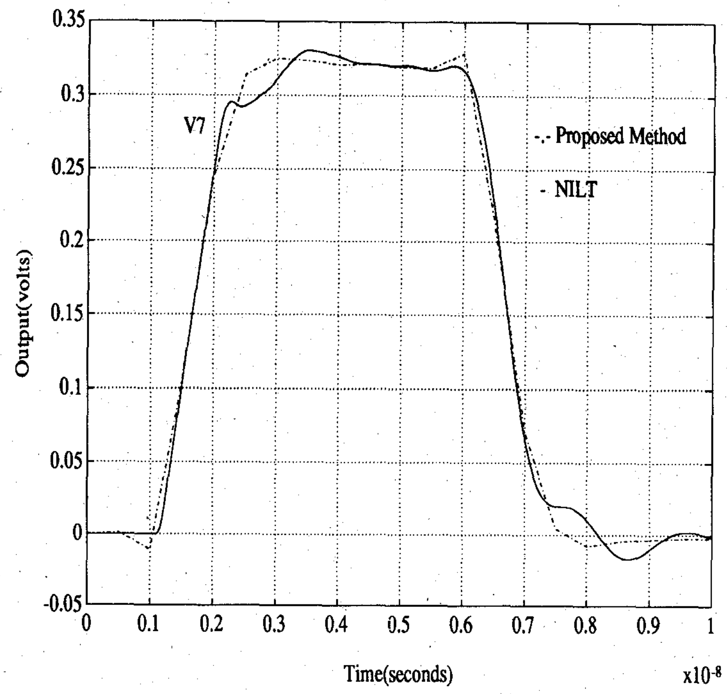

Fig. 7. Pulse response V_b of the network shown in Fig. 6.

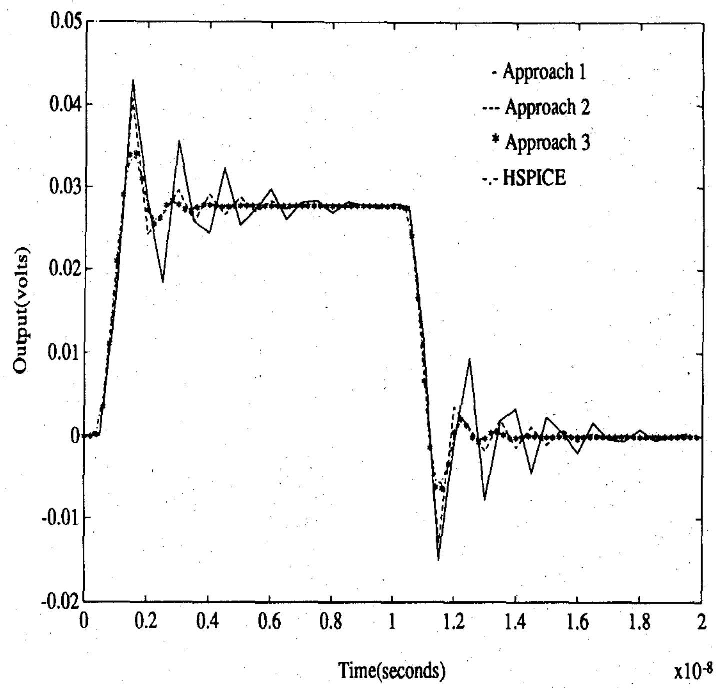

Fig. 8. Comparison between three different approaches for obtaining the response of the network shown in Fig. 2.

ing E_1 gives

$$E_1^{(n+1)} = \text{diagonal}\begin{bmatrix} -\frac{D}{2}\left(\left[e^{-\gamma_m D} - e^{\gamma_m D}\right]^{(n)}\gamma_m^{(1)}\right) - \\ \frac{D}{2}\sum_{r=1}^{n}\left[e^{-\gamma_m D} - e^{\gamma_m D}\right]^{(n-r)}\gamma_m^{(r+1)} \end{bmatrix}; \quad m = 1,\cdots,N. \tag{42}$$

Next, the derivatives of the eigenvalues and eigenvectors are evaluated using the solution of the eigenvalue problem

$$\left[\gamma_m^2 U - Z_L Y_L\right] S_m = 0 \tag{43}$$

where U is the identity matrix. The first derivative can be obtained as

$$\left[\gamma_m^2 U - Z_L Y_L \quad 2\gamma_m S_m\right]\begin{bmatrix} \frac{dS_m}{ds} \\ \frac{d\gamma_m}{ds} \end{bmatrix} = \left[\frac{d}{ds} Z_L Y_L\right] S_m. \tag{44}$$

Equation (44) represents a system of n equations with $(n+1)$ unknowns. To solve this system another equation normalizing the eigenvector $\boldsymbol{S}_m$ such that $\boldsymbol{S}_m^t \boldsymbol{S}_m = 1$ or

$$\boldsymbol{S}_m^t \frac{d\boldsymbol{S}_m}{ds} = 0. \tag{45}$$

is added to (44) yielding

$$\begin{bmatrix} \gamma_m^2 \boldsymbol{U} - \boldsymbol{Z}_L \boldsymbol{Y}_L & 2\gamma_m \boldsymbol{S}_m \\ \boldsymbol{S}_m^t & 0 \end{bmatrix} \begin{bmatrix} \dfrac{d\boldsymbol{S}_m}{ds} \\ \dfrac{d\gamma_m}{ds} \end{bmatrix} = \begin{bmatrix} \dfrac{d}{ds} \boldsymbol{Z}_L \boldsymbol{Y}_L \\ 0 \end{bmatrix} \boldsymbol{S}_m \tag{46}$$

Higher order derivatives are obtained recursively as

$$\boldsymbol{T}\begin{bmatrix} \boldsymbol{S}_m^{(n+1)} \\ \gamma_m^{(n+1)} \end{bmatrix} + \sum_{r=1}^{n} \binom{n}{r} \boldsymbol{T}^{(r)} \begin{bmatrix} \boldsymbol{S}_m^{(n-r+1)} \\ \gamma_m^{(n-r+1)} \end{bmatrix} = \begin{bmatrix} \sum_{r=0}^{n} \binom{n}{r} \left[\boldsymbol{Z}_p \boldsymbol{Y}_p \right]^{(n-r+1)} & \boldsymbol{S}_m^{(r)} \\ 0 & \end{bmatrix} \tag{47}$$

where

$$\boldsymbol{T} = \begin{bmatrix} \gamma_m^2 \boldsymbol{U} - \boldsymbol{Z}_L \boldsymbol{Y}_L & 2\gamma_m \boldsymbol{S}_m \\ \boldsymbol{S}_m^t & 0 \end{bmatrix}. \tag{48}$$

A similar method can be used to calculate the derivatives of other terms in (39) and (40) with respect to the derivatives of $\boldsymbol{S}_m$ and γ_m (47).

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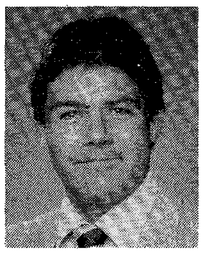

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